- Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

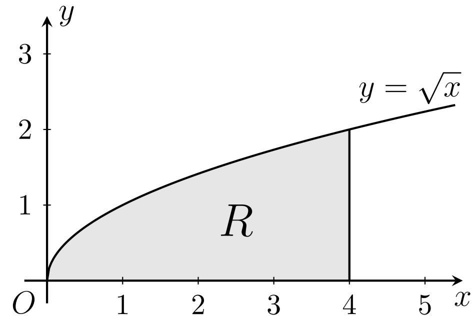
  (a) Find the area of the region R.

  (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

  (c) Find the volume of the solid generated when R is revolved about the x-axis.

  (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

Solutions:



(a)

$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3} \quad \text{or } 5.333$$

(b)

$$\int_0^h \sqrt{x}\,dx = \frac{8}{3} \quad \Bigg| \quad \int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$$

—or—

$$\frac{2}{3}h^{3/2} = \frac{8}{3} \quad \Bigg| \quad \frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$$

$$h = \sqrt[3]{16} \quad \text{or } 2.520 \quad \text{or } 2.519$$

(c)

$$V = \pi \int_0^4 (\sqrt{x})^2\,dx = \pi \cdot \frac{x^2}{2}\Big|_0^4 = 8\pi$$

$$\text{or } 25.133 \quad \text{or } 25.132$$

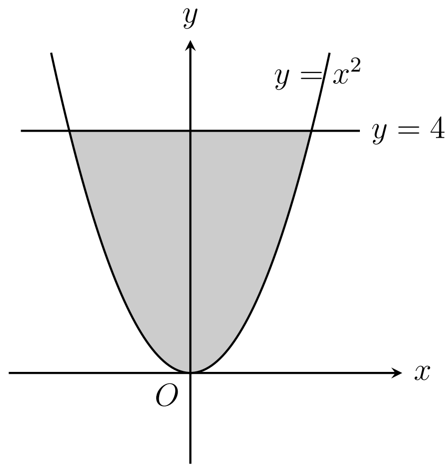
(d)

$$\pi \int_0^k (\sqrt{x})^2\,dx = 4\pi \quad \Bigg| \quad \pi \int_0^k (\sqrt{x})^2\,dx = \pi \int_k^4 (\sqrt{x})^2\,dx$$

—or—

$$\pi\frac{k^2}{2} = 4\pi \qquad \Bigg| \qquad \pi\frac{k^2}{2} = 8\pi - \pi\frac{k^2}{2}$$

$$k = \sqrt{8} \quad \text{or } 2.828$$



• The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x-axis.

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

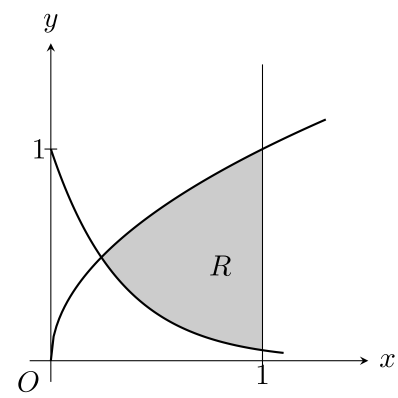
Solutions:

(a) $$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_{0}^{2} (4 - x^2)\,dx = 2\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{32}{3} = 10.666 \text{ or } 10.667$$

(b) $$\text{Volume} = \pi\int_{-2}^{2} (4^2 - (x^2)^2)\,dx = 2\pi\int_{0}^{2} (16 - x^4)\,dx = 2\pi\left[16x - \frac{x^5}{5}\right]_0^2 = \frac{256\pi}{5} = 160.849 \text{ or } 160.850$$

(c) $\pi\int_{-2}^{2}[(k - x^2)^2 - (k - 4)^2]\,dx = \frac{256\pi}{5}$

•



Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

Solutions:

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

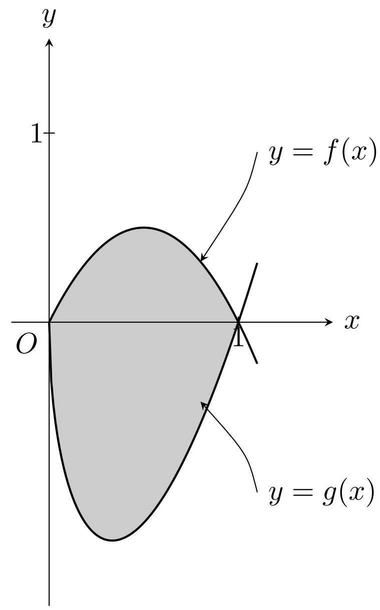
(a) $$\text{Area } = \int_T^1 (\sqrt{x} - e^{-3x})\, dx = 0.442 \text{ or } 0.443$$

(b) $$\text{Volume } = \pi \int_T^1 ((1 - e^{-3x})^2 - (1 - \sqrt{x})^2)\, dx = 0.453\pi \text{ or } 1.423 \text{ or } 1.424$$

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5(\sqrt{x} - e^{-3x})$

$$\text{Volume} = \int_T^1 5(\sqrt{x} - e^{-3x})^2\, dx = 1.554$$



- Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

Solutions:

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$

$$= \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right) dx = 1.133$$

(b) Volume $= \pi \int_0^1 \left((2-g(x))^2 - (2-f(x))^2\right) dx$

$$= \pi \int_0^1 \left(\left(2 - 3(x-1)\sqrt{x}\right)^2 - (2 - 2x(1-x))^2\right) dx$$

$$= 16.179$$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$

$$\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$$