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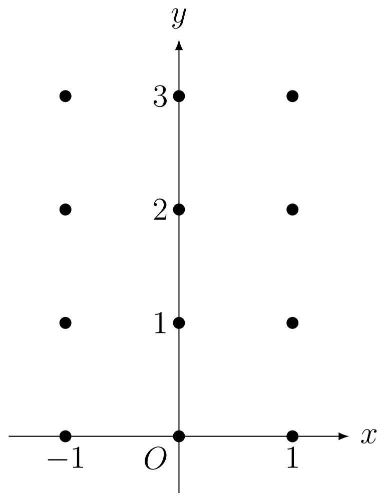
Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

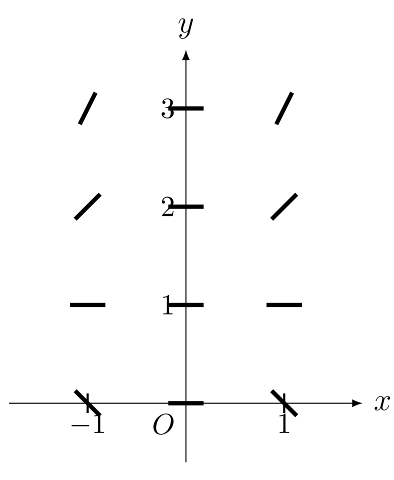


(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(*Note: Use the axes provided in the pink test booklet.*)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

(a)



(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

(c)

$$\frac{1}{y-1}dy = x^2dx$$
$$\ln|y-1| = \frac{1}{3}x^3 + C$$
$$|y-1| = e^C e^{\frac{1}{3}x^3}$$
$$y-1 = Ke^{\frac{1}{3}x^3}, K = \pm e^C$$
$$2 = Ke^0 = K$$
$$y = 1 + 2e^{\frac{1}{3}x^3}$$

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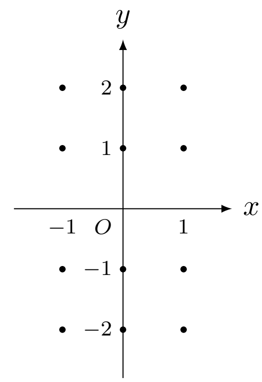
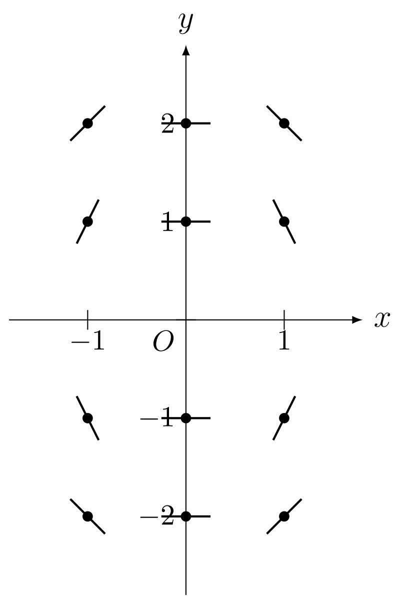
Consider the differential equation $\dfrac{dy}{dx} = -\dfrac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



(a)

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

(c)

$$\frac{dy}{dx} = -\frac{2x}{y}$$
$$y\,dy = -2x\,dx$$
$$\frac{y^2}{2} = -x^2 + C$$
$$\frac{1}{2} = -1 + C;\ C = \frac{3}{2}$$
$$y^2 = -2x^2 + 3$$

Since the particular solution goes through $(1, -1)$,
y must be negative.
Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

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Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

(b) $f(1.1) = 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$$\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0 \text{ on this interval.}$$

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

(c)

$$\frac{dy}{dx} = xy^3$$

$$\int \frac{1}{y^3}\,dy = \int x\,dx$$

$$-\frac{1}{2y^2} = \frac{x^2}{2} + C$$

$$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$$

$$y^2 = \frac{1}{\frac{5}{4} - x^2}$$

$$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad -\frac{\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$$