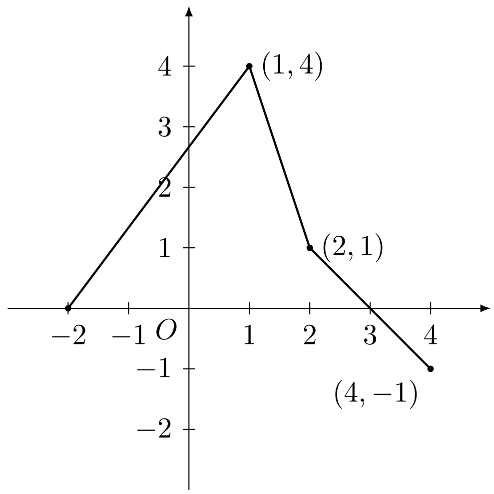
- The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

Solutions:

(a) $g(4) = \int_1^4 f(t)\,dt = \dfrac{3}{2} + 1 + \dfrac{1}{2} - \dfrac{1}{2} = \dfrac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\dfrac{1}{2}(12) = -6$

(b) $g'(1) = f(1) = 4$

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.
Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.
Since $g(-2) = -6$ and $g(4) = \dfrac{5}{2}$,
the absolute minimum value is -6.

(d) One; $x = 1$

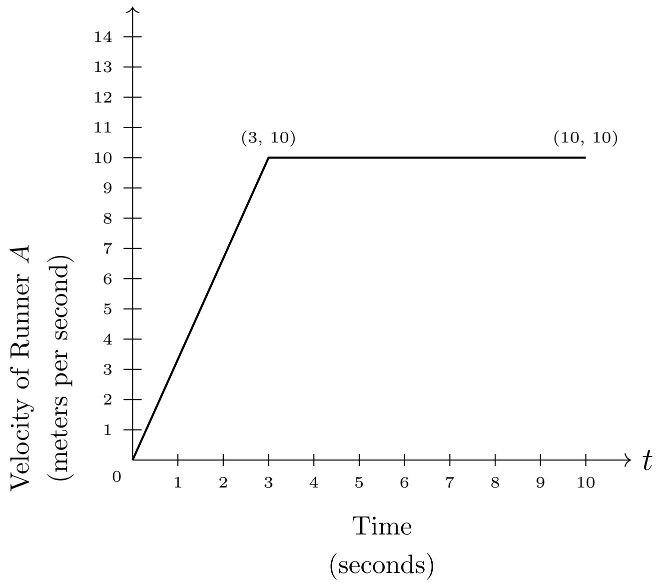
On $(-2, 1)$, $g''(x) = f'(x) > 0$

On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.

• Two runners, A and B, run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.



(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

Solutions:

(a) Runner A: velocity $= \frac{10}{3} \cdot 2 = \frac{20}{3}$

$$= 6.666 \text{ or } 6.667 \text{ meters/sec}$$

$$\text{Runner } B : v(2) = \frac{48}{7} = 6.857 \text{ meters/sec}$$

(b) Runner A: acceleration $= \frac{10}{3} = 3.333$ meters/sec^2

$$\text{Runner } B : a(2) = v'(2) = \left.\frac{72}{(2t+3)^2}\right|_{t=2}$$

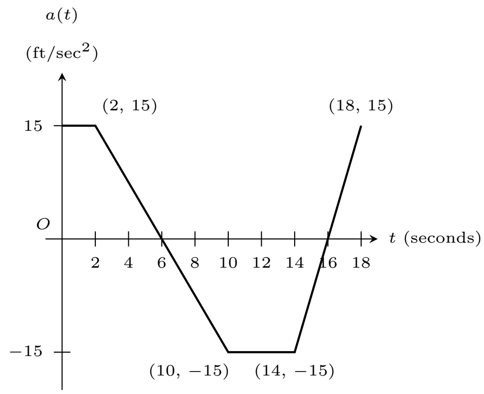
$$= \frac{72}{49} = 1.469 \text{ meters/sec}^2$$

(c) Runner A: distance $= \frac{1}{2}(3)(10) + 7(10) = 85$ meters

$$\text{Runner } B : \text{distance} = \int_0^{10} \frac{24t}{2t+3} dt = 83.336 \text{ meters}$$

(units) meters/sec in part (a), meters/sec^2 in part (b), and meters in part (c), or equivalent.

• A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

Solutions:

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.

(b) At time $t = 12$ because

$$v(12) - v(0) = \int_0^{12} a(t)\,dt = 0.$$

(c) The absolute maximum velocity is 115 ft/sec at $t = 6$.
The absolute maximum must occur at $t = 6$ or at an endpoint.

$$\begin{aligned} v(6) &= 55 + \int_0^6 a(t)\,dt \\ &= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0) \end{aligned}$$

$\int_6^{18} a(t)\,dt < 0$ so $v(18) < v(6)$

(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where

$$v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0.$$

• Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2-2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Solutions:

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

(c) $h'(4) = \dfrac{16-2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x-4)$

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

- A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

Solutions:

(a)

$$\begin{aligned} f'(x) &= 12x^2 + 2ax + b \\ f''(x) &= 24x + 2a \end{aligned}$$

$$\begin{aligned} f'(-1) &= 12 - 2a + b = 0 \\ f''(-2) &= -48 + 2a = 0 \end{aligned}$$

$$\begin{aligned} a &= 24 \\ b &= -12 + 2a = 36 \end{aligned}$$

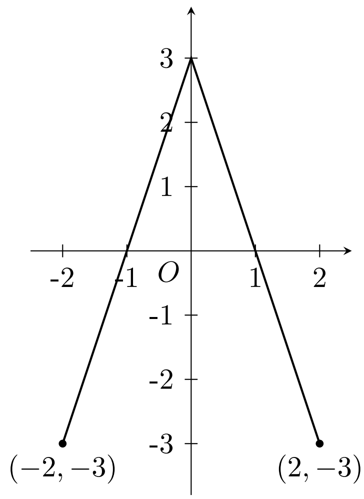
(b)

$$\begin{aligned} &\int_0^1 (4x^3 + 24x^2 + 36x + k)\,dx \\ &= x^4 + 8x^3 + 18x^2 + kx\Big|_{x=0}^{x=1} = 27 + k \end{aligned}$$

$$\begin{aligned} &27 + k = 32 \\ &k = 5 \end{aligned}$$

•

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.

Solutions:

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{3}{2}$
$g'(-1) = f(-1) = 0$
$g''(-1) = f'(-1) = 3$

(b) g is increasing on $-1 < x < 1$ because
$g'(x) = f(x) > 0$ on this interval.

(c) The graph of g is concave down on $0 < x < 2$
because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

(d)