Cylinder Volume and Surface Area Relationships

• A cylinder has a volume of 72π cubic centimeters and a height of 8 centimeters. What is the lateral surface area of the cylinder, in square centimeters? (Note: The lateral surface area of a cylinder is $2\pi rh$)

A) 18π

B) 36π

C) 48π

D) 72π

Solution:

The volume of a cylinder is $V = \pi r^2 h$.

Substitute the given values: $72\pi = \pi r^2(8)$.

Divide by 8π: $9 = r^2$, so $r = 3$.

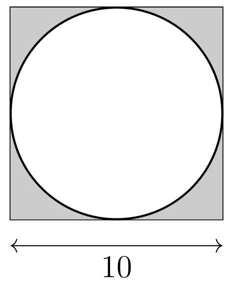
Lateral surface area is $L = 2\pi rh$.

$L = 2\pi(3)(8) = 48\pi$.

Correct Answer: C

Composite Figures and Shaded Area

• In the figure below, a circle is inscribed in a square with a side length of 10. What is the area of the shaded region?



A) $100 - 100\pi$

B) $100 - 25\pi$

C) $25 - 25\pi$

D) $10\pi - 100$

Solution:

Area of the square $= s^2 = 10^2 = 100$.

The diameter of the circle is equal to the side of the square (10), so the radius $r = 5$.

Area of the circle $= \pi r^2 = \pi(5)^2 = 25\pi$.

Shaded area = Area of square - Area of circle $= 100 - 25\pi$.

Correct Answer: B

Proportional Changes in Volume (Scaling)

• The volume of a right circular cone is V. If the radius of the cone is doubled and the height is reduced by half, what is the volume of the new cone in terms of V?

A) $\frac{1}{2}V$

B) V

C) $2V$

D) $4V$

Solution:
Original Volume: $V = \dfrac{1}{3}\pi r^2 h$.
New dimensions: $r_{new} = 2r$ and $h_{new} = \dfrac{1}{2}h$.
New Volume $V_{new} = \dfrac{1}{3}\pi(2r)^2(\dfrac{1}{2}h) = \dfrac{1}{3}\pi(4r^2)(\dfrac{1}{2}h)$.
Simplify: $V_{new} = 2 \times (\dfrac{1}{3}\pi r^2 h) = 2V$.

Correct Answer: C

• A cylinder has a radius of 3 and a height of 8. A second cylinder has a radius of 6 and a height of 2. What is the ratio of the volume of the first cylinder to the volume of the second cylinder?

A) $1:1$

B) $1:2$

C) $2:3$

D) $3:4$

Solution:

$$V_1 = \pi(3)^2(8) = 72\pi \qquad V_2 = \pi(6)^2(2) = 72\pi$$
$$\frac{V_1}{V_2} = \frac{72\pi}{72\pi} = \boxed{1:1}$$

Even though the radii differ, the volumes are equal because the larger radius is offset by the smaller height.

Correct Answer: A

Spheres (Volume and Surface Area)

• A sphere has a volume of 36π cubic inches. What is the surface area of the sphere, in square inches? (Note: $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$)

A) 12π

B) 18π

C) 36π

D) 72π

Solution:

From given volume $36\pi = \dfrac{4}{3}\pi r^3$ we can find radius.

Multiply both sides of the equation above by by $\dfrac{3}{4\pi}$ we get $27 = r^3$, therefore $r = 3$.

Surface Area $= 4\pi(3)^2 = 4\pi(9) = 36\pi$.

Correct Answer: C

• A sphere is inscribed in a cylinder (the sphere touches the top, bottom, and lateral surface of the cylinder). If the radius of the sphere is r, what fraction of the cylinder's volume is occupied by the sphere?

A) $\dfrac{1}{2}$

B) $\dfrac{2}{3}$

C) $\dfrac{3}{4}$

D) $\dfrac{\pi}{4}$

Solution:
Since the sphere is inscribed, the cylinder has radius r and height $= 2r$ (diameter of sphere).

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad V_{\text{cylinder}} = \pi r^2 \cdot 2r = 2\pi r^3$$

$$\frac{V_{\text{sphere}}}{V_{\text{cylinder}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{4/3}{2} = \boxed{\frac{2}{3}}$$

This is Archimedes' famous result: a sphere occupies exactly $\frac{2}{3}$ of its circumscribed cylinder.

Correct Answer: B

Algebraic Modeling of Volume

• A rectangular prism has a length of x, a width of $x-2$, and a height of 5. If the volume of the prism is 120, what is the value of x?

A) 4

B) 6

C) 8

D) 10

Solution:

$V = l \cdot w \cdot h = x(x-2)(5) = 120.$

Divide by 5: $x(x-2) = 24.$

$x^2 - 2x - 24 = 0.$

Factor: $(x-6)(x+4) = 0.$

Since x must be positive, $x = 6.$

Correct Answer: B

• The dimensions of a rectangular box are in the ratio $1 : 2 : 3$. If the volume of the box is 162 cubic centimeters, what is the surface area of the box, in square centimeters?

A) 126

B) 162

C) 198

D) 216

Solution:
Let the dimensions of rectangular box be x, $2x$, and $3x$. Then:

$$V = x \cdot 2x \cdot 3x = 6x^3 = 162 \implies x^3 = 27 \implies x = 3$$

The dimensions of the box are 3, 6, and 9.

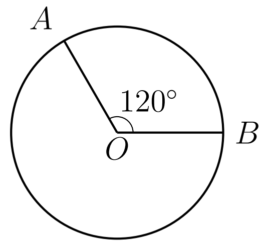
Surface area:

$$SA = 2(lw + lh + wh) = 2\big[(3)(6) + (6)(9) + (3)(9)\big] = 2[18 + 54 + 27] = 2(99) = \boxed{198}$$

Correct Answer: C

Arc Length and Sector Area

• In the figure below, the area of the sector of the circle with center O is 12π. If the measure of $\angle AOB$ is $120°$, what is the length of arc AB?



A) 2π

B) 4π

C) 6π

D) 8π

Solution:

Sector Area $= \dfrac{\theta}{360}\pi r^2$.

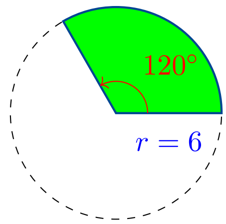
$12\pi = \dfrac{120}{360}\pi r^2 = \dfrac{1}{3}\pi r^2$.

$36 = r^2$, so $r = 6$.

Arc Length $= \dfrac{\theta}{360}2\pi r = \dfrac{1}{3}(2\pi \cdot 6) = 4\pi$.

Correct Answer: B

- A sector of a circle has a central angle of 120° and a radius of 6. What is the area of the sector?



A) 6π

B) 8π

C) 12π

D) 18π

Solution:
The sector area formula is:

$$A_{\text{sector}} = \frac{\theta}{360^\circ} \cdot \pi r^2 = \frac{120}{360} \cdot \pi(6)^2 = \frac{1}{3} \cdot 36\pi = \boxed{12\pi}$$

Correct Answer: C

Volume of Pyramids/ Cone

• A pyramid has a square base with an area of 36 square inches. If the height of the pyramid is 10 inches, what is the volume of the pyramid in cubic inches?

A) 120

B) 180

C) 360

D) 1,080

Solution:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base.

$V = \frac{1}{3}(36)(10)$.

$V = 12 \times 10 = 120$.

Correct Answer: A

• A cone has a volume of 48π cubic inches. If the height of the cone is 9 inches, what is the radius of the base of the cone, in inches?

A) 2

B) 4

C) 6

D) 8

Use the cone volume formula and solve for r:

$$V = \frac{1}{3}\pi r^2 h \implies 48\pi = \frac{1}{3}\pi r^2(9) = 3\pi r^2$$

$$r^2 = \frac{48\pi}{3\pi} = 16 \implies r = \boxed{4}$$

Correct Answer: B

Circumference and Area Relationships

• The area of a circle is A. If the circumference of the circle is tripled, what is the area of the new circle in terms of A?

A) $3A$

B) $6A$

C) $9A$

D) $27A$

Solution:

$C = 2\pi r$. If C is tripled, r must also be tripled.

New radius $r' = 3r$.

New Area $A' = \pi(3r)^2 = 9\pi r^2$.

Since the original area was $A = \pi r^2$, the new area is $9A$.

Correct Answer: C

• A circle has a circumference of 20π. What is the area of the circle?

A) 10π

B) 20π

C) 100π

D) 400π

Solution:
Use the circumference formula:

$$C = 2\pi r = 20\pi \implies r = 10$$

Then compute the area:

$$A = \pi r^2 = \pi(10)^2 = \boxed{100\pi}$$

Correct Answer: C

Volume Displacement and Transfer

• A container is in the shape of a right rectangular prism with a base of 4 feet by 5 feet. It is filled with water to a height of 2 feet. If this water is poured into an empty cylindrical tank with a radius of 3 feet, what will be the height of the water in the cylindrical tank?

A) $\frac{20}{3\pi}$

B) $\frac{40}{3\pi}$

C) $\frac{20}{9\pi}$

D) $\frac{40}{9\pi}$

Solution:

Volume of water in prism $= lwh = 4 \times 5 \times 2 = 40$ cubic feet.

Volume of water in cylinder $= \pi r^2 h = \pi(3)^2 h = 9\pi h$.

Set volumes equal: $40 = 9\pi h$.

Solve for h: $h = \frac{40}{9\pi}$.

Correct Answer: D

Area of Special Triangles

- An equilateral triangle has a side length of 6. What is the area of this triangle?

A) 9

B) $9\sqrt{3}$

C) $18\sqrt{3}$

D) $36\sqrt{3}$

Solution:

The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$.

Substitute $s = 6$: $Area = \frac{6^2\sqrt{3}}{4} = \frac{36\sqrt{3}}{4}$.

$Area = 9\sqrt{3}$.

Correct Answer: B

Problems adapted from the College Board SAT Question Bank and released SAT practice tests.