

• Ms. Fey is a manager at a restaurant. To improve the dining experience for her customers, she uses a digital music service to create a playlist of songs that will be played in the restaurant. The playlist contains 1,000 songs and consists of four different types of music in the following quantities: 200 country songs, 400 pop songs, 100 rock songs, and 300 jazz songs. The digital music service will select songs at random from the playlist to be played in the restaurant. Any song can be replayed at any time.

(A)

- i. Suppose one song is selected at random to be played. What is the probability that the song is a rock song? Show your work.
- ii. Suppose two songs are selected at random to be played. What is the probability that both songs are rock songs? Show your work.

(B)

In every one-hour period, 20 songs will be played at random and any song can be replayed at any time. Ms. Fey is interested in how many rock songs will be played in a typical one-hour period.

- i. Define the random variable of interest to Ms. Fey, and state how the random variable is distributed.
- ii. What is the expected value for the random variable in part B (i)? Show your work.

(C)

Recall that in every one-hour period, 20 songs will be played at random and any song can be replayed at any time.

- i. Determine the probability that 4 or more rock songs in a particular one-hour period will be played. Show your work.
- ii. Suppose 4 rock songs are played during a particular one-hour period. Does this provide strong evidence that the song selection process was not truly random? Justify your answer without performing an inference procedure.

*Solution:*

(A)

i.  $P(\text{Rock Song}) = \frac{100}{1,000} = 0.10$

ii.  $P(\text{Both Rock Songs}) = (0.10)(0.10) = 0.01$

(B)

i. Let the random variable of interest,  $X$ , represent the number of the 20 songs played in one hour that are rock songs. It is stated that any song can be replayed at any time, which establishes that each rock song has probability  $\frac{100}{1,000} = 0.10$  of being selected each hour and each song is independent from every other song. Therefore,  $X$  has a binomial distribution with  $n = 20$  independent trials and probability of success  $p = 0.10$  for each trial.

ii. The expected value for the number of rock songs played in one hour is  $np = 20(0.10) = 2$  songs.

(C)

i. The probability that in a particular hour 4 or more rock songs will be played is  $P(X \geq 4) = 1 - P(X \leq 3)$

$$P(X \geq 4) = 1 - \binom{20}{0}(0.10)^0(0.90)^{20} + \binom{20}{1}(0.10)^1(0.90)^{19} \\ + \binom{20}{2}(0.10)^2(0.90)^{18} + \binom{20}{3}(0.10)^3(0.90)^{17}$$

$$P(X \geq 4) = 1 - 0.867 = 0.133.$$

ii. No, the probability that 4 or more rock songs would be played in an hour is 0.133, which is high enough to be reasonably attributed to chance alone. This probability is not small enough to provide evidence that the selection process was not truly random.

• In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(A) Sarah, a player, will collect and open geodes until a red crystal is found.

- i. Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.
- ii. Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(B) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let  $Y$  = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable  $Y$ .

Number of geodes Conrad will open, $y$	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736		

- i. Calculate  $P(Y = 3)$ . Show your work.
- ii. Calculate  $P(Y = 4)$ . Show your work.

(C) Consider the table and your results from part (b).

- i. Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.
- ii. Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

*Solution:*

(A) Let  $G$  represent the number of geodes a player opens until finding a red crystal.  $G$  follows a geometric distribution with  $p = 0.08$ .

$$(i) \mu = E(x) = \frac{1}{p} \Rightarrow \mu = E(G) = \frac{1}{0.08} \approx 12.5 \text{ geodes}$$

$$(ii) \text{Var}(x) = \sigma^2 = \frac{1-p}{p^2}$$

$$\sigma_G = \frac{\sqrt{1-0.08}}{0.08} \approx 11.99 \text{ geodes}$$

- (B) (i)  $P(X = n) = (1-p)^{n-1}(p)$   
 $P(Y = 3) = (0.92)^2(0.08) \approx 0.067712$
- (ii)  $P(Y = 4) = 1 - P(Y = 1 \text{ or } 2 \text{ or } 3)$   
 $\approx 1 - (0.08 + 0.0736 + 0.067712)$   
 $\approx 0.778688$

OR

If Conrad opens 4 geodes, then he either finds no red geodes or he finds a red geode on the fourth one he opens; therefore,  $P(Y = 4) = (0.92)^4 + (0.92)^3(0.08)$   
 $\approx 0.778688$ .

#### Additional Notes:

- A response may satisfy component 2 or component 4 by the following or a combination of the following:
  - **Probability formula:** Displaying a correct formula for computing the geometric probability, such as:
    - \*  $(1 - 0.08)^2(0.08)$  or  $(0.92)^2(0.08)$  for part (b-i)
  - **Calculator function syntax:** Using calculator function notation with the correct value of the parameter identified, such as:
    - \*  $\text{geompdf}(p = 0.08, x = 3)$  for part (b-i)
    - \*  $1 - \text{geomcdf}(p = 0.08, x = 3)$  for part (b-ii)
    - \*  $\text{binompdf}(n = 4, p = 0.08, x = 0) + \text{geompdf}(p = 0.08, x = 4)$  for part (b-ii)
    - \*  $\text{binompdf}(n = 4, p = 0.92, x = 4) + \text{geompdf}(p = 0.08, x = 4)$  for part (b-ii)
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

- (C) (i)  $\mu = E(Y)$   
 $\approx (1)(0.08) + (2)(0.0736) + (3)(0.0677) + (4)(0.778688)$   
 $\approx 0.08 + 0.1472 + 0.2031 + 3.1148$   
 $\approx 3.545 \text{ geodes.}$
- (ii) The mean of 3.545 geodes is the average number of geodes that result from a long run of many, many trials of opening randomly selected geodes and counting the number opened until either a red geode is found or the fourth geode is opened.

• Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable,  $X$ , represent the dollar value of the cash prize in a bath fizzy. The probability distribution of  $X$  is shown in the table.

Cash prize, $x$	\$1	\$5	\$10	\$20	\$50	\$100
Probability of cash prize, $P(X = x)$	0.2	0.05	0.05	0.05	0.01	0.01

- (A) Based on the probability distribution of  $X$ , answer the following. Show your work.
- Calculate the proportion of bath fizzies that contain \$1.
  - Calculate the proportion of bath fizzies that contain at least \$10.
- (B) Based on the probability distribution of  $X$ , calculate the probability that a randomly selected bath fizzy contains \$100, given that it contains at least \$10. Show your work.
- (C) Based on the probability distribution of  $X$ , calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work.
- (D) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

*Solution:*

(A) The random variable  $X$  is the dollar value of the cash prize in a bath fizzy.

(i) The proportion of bath fizzies containing \$1 is equal to the  $P(X = \$1)$  and

$$P(X = \$1) = 1 - (0.2 + 0.05 + 0.05 + 0.01 + 0.01) = 0.68.$$

(ii) The proportion of bath fizzies that contain at least \$10 is equal to the  $P(X \geq \$10)$  and

$$P(X \geq \$10) = 0.05 + 0.05 + 0.01 + 0.01 = 0.12.$$

(B) Given a bath fizzy contains at least \$10, then the probability that it contains \$100 is

$$P(X = \$100 \mid X \geq \$10) = \frac{0.01}{0.12} \approx 0.0833.$$

(C) The expected value of the distribution of  $X$  is

$$\begin{aligned} E(X) &= 1(0.68) + 5(0.2) + 10(0.05) \\ &\quad + 20(0.05) + 50(0.01) + 100(0.01) = \$4.68. \end{aligned}$$

The expected value is the mean of the cash prizes that result from the long run of many, many trials of randomly selecting bath fizzies and determining the amount each contains.

(D) The expected value of the distribution of  $X$  in euros is  $4.68(0.89) \simeq 4.17$  euros.

• A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable  $A$  represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

(A) A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

(B) The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

- (i) Define the random variable of interest for the warehouse manager and state how the random variable is distributed.
- (ii) Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

To reduce the number of crates rejected by the beauty supply warehouse manager, the manufacturing company is considering adjusting the programming of the filling machine so that the amount of shampoo in each bottle is normally distributed with mean 0.56 liter and standard deviation 0.03 liter.

(C) Would you recommend that the manufacturing company use the original programming of the filling machine or the adjusted programming of the filling machine? Provide a statistical justification for your choice.

*Solutions:*

(A) Random variable  $A$ , which represents the amount of shampoo in a randomly selected bottle, follows a normal distribution with mean 0.6 liter and standard deviation 0.04 liter. Then, the probability that a randomly selected bottle is underfilled is

$$P(A < 0.5) = P\left(Z < \frac{0.5 - 0.6}{0.04} = -2.5\right) \approx 0.0062.$$

(B)

(i) The random variable of interest,  $X$ , is the number of underfilled bottles in a box of 10 bottles. The distribution of  $X$  is binomial with parameters  $n = 10$  and  $p = 0.0062$ .

(ii) The crate will be rejected by the warehouse if two or more underfilled bottles are found in the box. The probability of that is

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - \binom{10}{1}(0.0062)^1(0.9938)^9 \\ &\quad - \binom{10}{0}(0.0062)^0(0.9938)^{10} \\ &\approx 0.0017. \end{aligned}$$

(C) The company should use the original programming for the filling machine. For the original programming of the filling machine, the probability of an underfilled bottle is

$$\begin{aligned} P(A < 0.5) &= P\left(Z < \frac{0.5 - 0.60}{0.04}\right) \\ &= P(Z < -2.5) \approx 0.0062. \end{aligned}$$

For the adjusted programming of the filling machine, the probability of an underfilled bottle is

$$\begin{aligned} P(A < 0.5) &= P\left(Z < \frac{0.5 - 0.56}{0.03}\right) \\ &= P(Z < -2.0) \approx 0.02275. \end{aligned}$$

Because the probability of an underfilled bottle is greater for the adjusted programming, this would result in more rejected shipments. The company should continue with the original machine programming.

- To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week.
- (A) Consider the probability that a particular employee receives at least one gift card in a 52-week year.
- (i) Define the random variable of interest and state how the random variable is distributed.
  - (ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.
- (B) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.
- (C) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

*Solutions:*

- (A) (i) Let the random variable of interest  $X$  represent the number of gift cards that a particular employee receives in a 52-week year. Because each employee has probability  $\frac{1}{200} = 0.005$  of being selected each week to receive a gift card and each week's selection is independent from every other week,  $X$  has a binomial distribution with  $n = 52$  repeated independent trials and probability of success  $p = 0.005$  for each trial.
- (ii) The probability that a particular employee receives at least one gift card in a 52-week year is:

$$\begin{aligned}P(X \geq 1) &= 1 - P(X = 0) \\&= 1 - \binom{52}{0} (0.005)^0 (0.995)^{52} \\&= 1 - 0.7705 \\&= 0.2295\end{aligned}$$

- (B) The expected value for the number of gift cards a particular employee will receive in a 52-week year is  $np = 52(0.005) = 0.26$ . If the random process of selecting one employee each week to receive a gift card is repeated for a very large number of years, each employee can expect to receive about 0.26 gift cards per year, on average, or about one gift card every four years.
- (C) No, Agatha's experience does not constitute strong evidence that the selection process was not truly random. In fact, it is quite likely (probability  $= (0.995)^{52} \simeq 0.7705$ ) that a particular employee will fail to receive a gift card for an entire 52-week year.

- A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (A) One person from those surveyed will be selected at random.
- What is the probability that the person selected will be someone whose response is never and who is a woman?
  - What is the probability that the person selected will be someone whose response is never or who is a woman?
  - What is the probability that the person selected will be someone whose response is never given that the person is a woman?
- (B) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.
- (C) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

*Solutions:*

(A):

$$(i) P(\text{never and woman}) = 0.0636$$

$$(ii) P(\text{never or woman}) = P(\text{never}) + P(\text{woman}) - P(\text{never and woman}) \\ = 0.12 + 0.53 - 0.0636 \\ = 0.5864$$

$$(iii) P(\text{never} \mid \text{woman}) = \frac{P(\text{never and woman})}{P(\text{woman})} = \frac{0.0636}{0.53} = 0.12$$

(B): Yes, the event of being a person who responds never is independent of the event of being a woman because

$$P(\text{never} \mid \text{woman}) = P(\text{never}) = 0.12.$$

(C): Define  $X$  as the number of people in a random sample of five people who always take their medicine as prescribed. Then  $X$  has a binomial distribution with  $n = 5$  and  $p = 0.54$ , and

$$P(X \geq 4) = \binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0 \approx 0.19557 + 0.04592 \approx 0.24149.$$

• A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of \$50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.

- (A) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.
- (B) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?
- (C) The company has a gain of \$50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of \$150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

*Solutions:*

(A): The 25th percentile of the standard normal distribution is  $-0.6745$ . Consequently the 25th percentile of a normal distribution with mean 30 months and standard deviation 8 months is  $30 + 8(-0.6745) = 24.6$  months.

(B) The probability that a randomly selected customer will need to request a replacement because the battery fails within 24 months from the date of purchase is

$$P(\text{life span} \leq 24 \text{ months}) = P\left(Z \leq \frac{24 - 30}{8}\right) = P(Z \leq -0.75) \approx 0.2266.$$

(C) The company's expected gain for each warranty purchased is

$$\begin{aligned} &(\$50) \times P(\text{life span} > 24 \text{ months}) + (-\$150) \times P(\text{life span} \leq 24 \text{ months}) \\ &= (\$50) \times (0.7734) + (-\$150) \times (0.2266) \approx \$4.68. \end{aligned}$$

*Solutions:*

(A) The 25th percentile of the standard normal distribution is  $-0.6745$ . Consequently the 25th percentile of a normal distribution with mean 30 months and standard deviation 8 months is  $30 + 8(-0.6745) = 24.6$  months.

(B) The probability that a randomly selected customer will need to request a replacement because the battery fails within 24 months from the date of purchase is

$$P(\text{life span} \leq 24 \text{ months}) = P\left(Z \leq \frac{24 - 30}{8}\right) = P(Z \leq -0.75) \approx 0.2266.$$

(C) The company's expected gain for each warranty purchased is

$$\begin{aligned} &(\$50) \times P(\text{life span} > 24 \text{ months}) + (-\$150) \times P(\text{life span} \leq 24 \text{ months}) \\ &= (\$50) \times (0.7734) + (-\$150) \times (0.2266) \approx \$4.68. \end{aligned}$$

• Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.

- (A) What is the probability that a randomly selected child born in the region is left-handed?
- (B) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?
- (C) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

*Solution*

(A) Let  $L$  denote left-handed,  $M$  denote multiple birth, and  $S$  denote single birth. The probability that a randomly selected child born in the region is left-handed is:

$$P(L) = P(M)P(L | M) + P(S)P(L | S) = (0.035)(0.22) + (0.965)(0.11) = 0.0077 + 0.10615 = 0.11385.$$

(B) From part (a),  $P(L) = 0.11385$ . Therefore,

$$P(M | L) = \frac{P(L \text{ and } M)}{P(L)} = \frac{(0.035)(0.22)}{0.11385} = \frac{0.0077}{0.11385} \approx 0.0676.$$

(C) Let  $X$  represent the number of children who are left-handed in a random sample of 20 children from the region.  $X$  has a binomial distribution with  $n = 20$  and  $p = 0.11385$  (found in part (a)). Using the binomial distribution,

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - \binom{20}{0}(0.11385)^0(0.88615)^{20} - \binom{20}{1}(0.11385)^1(0.88615)^{19} - \binom{20}{2}(0.11385)^2(0.88615)^{18} \\ &\approx 1 - 0.598 \approx 0.402. \end{aligned}$$

- Consider an experiment in which two men and two women will be randomly assigned to either a treatment group or a control group in such a way that each group has two people. The people are identified as Man 1, Man 2, Woman 1, and Woman 2. The six possible arrangements are shown below.

Arrangement A	
Treatment	Control
Man 1	Woman 1
Man 2	Woman 2

Arrangement B	
Treatment	Control
Man 1	Man 2
Woman 1	Woman 2

Arrangement C	
Treatment	Control
Man 1	Man 2
Woman 2	Woman 1

Arrangement D	
Treatment	Control
Woman 1	Man 1
Woman 2	Man 2

Arrangement E	
Treatment	Control
Man 2	Man 1
Woman 2	Woman 1

Arrangement F	
Treatment	Control
Man 2	Man 1
Woman 1	Woman 2

Two possible methods of assignment are being considered: the sequential coin flip method, as described in part (a), and the chip method, as described in part (b). For each method, the order of the assignment will be Man 1, Man 2, Woman 1, Woman 2.

- (A) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.

- (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	B	C	D	E	F
Probability						

- (ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

The six arrangements are repeated below.

Arrangement A	
Treatment	Control
Man 1	Woman 1
Man 2	Woman 2

Arrangement B	
Treatment	Control
Man 1	Man 2
Woman 1	Woman 2

Arrangement C	
Treatment	Control
Man 1	Man 2
Woman 2	Woman 1

Arrangement D	
Treatment	Control
Woman 1	Man 1
Woman 2	Man 2

Arrangement E	
Treatment	Control
Man 2	Man 1
Woman 2	Woman 1

Arrangement F	
Treatment	Control
Man 2	Man 1
Woman 1	Woman 2

- (B) For the chip method, two chips are marked “treatment” and two chips are marked “control.” Each person selects one chip at random without replacement.
- (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	B	C	D	E	F
Probability						

- (ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?
- (C) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatment? Justify your answer.

*Solution*

(A)

(i) Let T (tail) represent being assigned to the treatment group and H (head) represent being assigned to the control group for each coin flip. The process stops when either the treatment group or the control group has two members. The outcomes and their probabilities are as follows.

Arrangement	A	B	C	D	E	F
Chip outcomes	TT	THT	THH	HH	HTH	HTT
Calculation	$(\frac{1}{2})(\frac{1}{2})$	$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$	$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$	$(\frac{1}{2})(\frac{1}{2})$	$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$	$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

(B)

(i) Let T represent being assigned to the treatment group and C represent being assigned to the control group for each chip drawn. The process stops when either the treatment group or the control

group has two members. The probabilities differ from the coin flip method because chips are drawn *without replacement*. The outcomes and their probabilities are as follows.

Arrangement	A	B	C	D	E	F
Chip outcomes	TT	TCT	TCC	CC	CTC	CTT
Calculation	$\binom{2}{4} \binom{1}{3}$	$\binom{2}{4} \binom{2}{3} \binom{1}{2}$	$\binom{2}{4} \binom{2}{3} \binom{1}{2}$	$\binom{2}{4} \binom{1}{3}$	$\binom{2}{4} \binom{2}{3} \binom{1}{2}$	$\binom{2}{4} \binom{2}{3} \binom{1}{2}$
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$$

(C)

Use the chip method. The chip method gives equal probability to all possible arrangements, but the coin method does not, as shown in the tables from parts (a-i) and (b-i). Furthermore, the coin method is more likely to result in imbalanced treatment groups with regard to students and teachers, based on the probabilities in parts (a-ii) and (b-ii). If food preferences for teachers are different than for students, the imbalance is a problem. For example, if one treatment group consists entirely of students, it would be impossible to know if a difference in the response variable is due to the treatment (type of meal) or the role of the person at the school (teacher or student).

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*Problems adapted from the College Board released tests.*